Lesson 3: Identical Triangles

☐ I can use triangle correspondence to recognize when two triangles math identically.
☐ I can use precise notation to show triangle correspondence and describe corresponding angles & sides.

Opening

When studying triangles, it is essential to be able to communicate about the parts of a triangle without any confusion. The following terms are used to identify particular angles or sides:

- between
- adjacent to
- opposite to
- included [side/angle]

Exercises 1–7

Use the figure $\triangle ABC$ to fill in the following blanks.

1. $\angle A$ is _____ sides $\overline{AB}$ and $\overline{AC}$.

2. $\angle B$ is _____ side $\overline{AB}$ and to side $\overline{BC}$.

3. Side $\overline{AB}$ is _____ $\angle C$.

4. Side _____ is the included side of $\angle B$ and $\angle C$.

5. $\angle$ _____ is opposite to side $\overline{AC}$.

6. Side $\overline{AB}$ is between $\angle$ _____ and $\angle$ _____.

7. What is the included angle of sides $\overline{AB}$ and $\overline{BC}$ _____
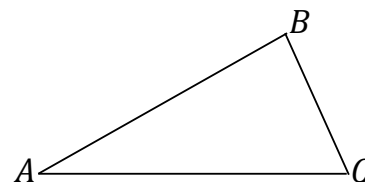
Now that we know what to call the parts within a triangle, we consider how to discuss two triangles. We need to compare the parts of the triangles in a way that is easy to understand. To establish some alignment between the triangles, we pair up the vertices of the two triangles. We call this a *correspondence*. Specifically, a correspondence between two triangles is a pairing of each vertex of one triangle with one (and only one) vertex of the other triangle. A correspondence provides a systematic way to compare parts of two triangles.
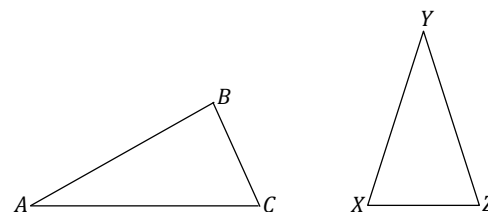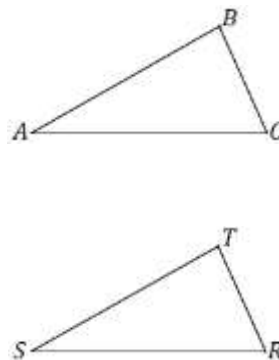
Figure 1

With a correspondence in place, comparisons can be made about corresponding sides and corresponding angles. The following are corresponding vertices, angles, and sides for the triangle correspondence $\triangle ABC \leftrightarrow \triangle YXZ$. Complete the missing correspondences.

Vertices:	$A \leftrightarrow Y$	$B \leftrightarrow$	$C \leftrightarrow$
Angles:	$\angle A \leftrightarrow \angle Y$	$\angle B \leftrightarrow$	$\angle C \leftrightarrow$
Sides:	$\overline{AB} \leftrightarrow \overline{YX}$	$\overline{BC} \leftrightarrow$	$\overline{CA} \leftrightarrow$

Example 1

Given the following triangle correspondences, use double arrows to show the correspondence between vertices, angles, and sides.

Triangle Correspondence	$\triangle ABC \leftrightarrow \triangle STR$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	

Examine Figure 2. By simply looking, it is impossible to tell the two triangles apart unless they are labeled. They look exactly the same (just as identical twins look the same). One triangle could be picked up and placed on top of the other.

Two triangles are identical if there is a triangle correspondence so that corresponding sides and angles of each triangle are equal in measurement. In Figure 2, there is a correspondence that will match up equal sides and equal angles, $\triangle ABC \leftrightarrow \triangle XYZ$; we can conclude that $\triangle ABC$ is identical to $\triangle XYZ$. This is not to say that we cannot find a correspondence in Figure 2 so that unequal sides and unequal angles are matched up, but there certainly is one correspondence that will match up angles with equal measurements and sides of equal lengths, making the triangles identical.
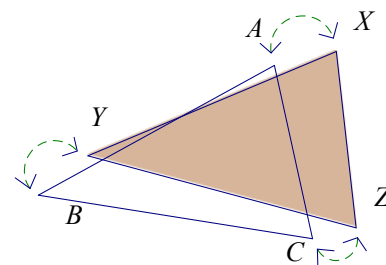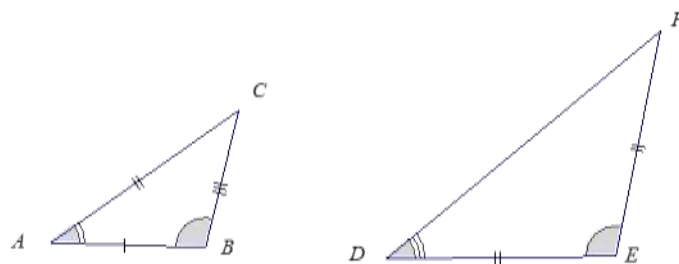
Figure 2

In discussing identical triangles, it is useful to have a way to indicate those sides and angles that are equal. We mark sides with tick marks and angles with arcs if we want to draw attention to them. If two angles or two sides have the same number of marks, it means they are equal.

In this figure, $AC = DE = EF$, and $\angle B = \angle E$.

Example 2

Two identical triangles are shown below. Give a triangle correspondence that matches equal sides and equal angles.
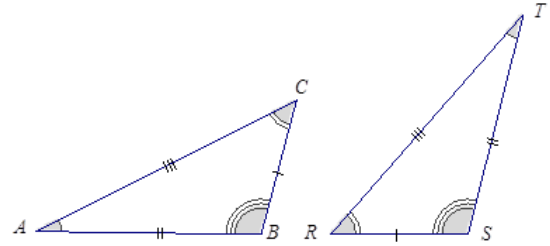
Exercise 8

8.   Sketch two triangles that have a correspondence. Describe the correspondence in symbols or words. Have a partner check your work.

Lesson Summary

- Two triangles and their respective parts can be compared once a correspondence has been assigned to the two triangles. Once a correspondence is selected, corresponding sides and corresponding angles can also be determined.

- Double arrows notate corresponding vertices. Triangle correspondences can also be notated with double arrows.

- Triangles are identical if there is a correspondence so that corresponding sides and angles are equal.

- An equal number of tick marks on two different sides indicates the sides are equal in measurement. An equal number of arcs on two different angles indicates the angles are equal in measurement.

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